# Otoczka wypukła

## (ang. Convex Hull)

2026

# The convex hull problem

- Otoczka wypukła zbioru punktów Q to najmniejszy wielokąt wypukły P taki, że każdy punkt ze zbioru Q leży albo na brzegu P, albo w jego wnętrzu.

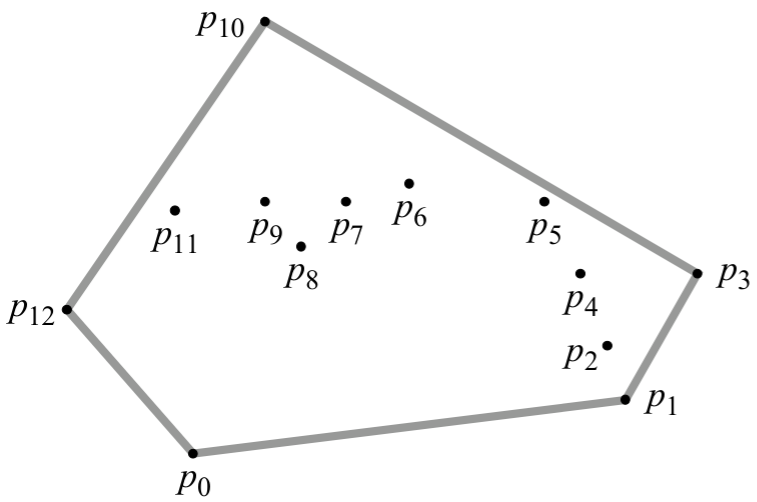


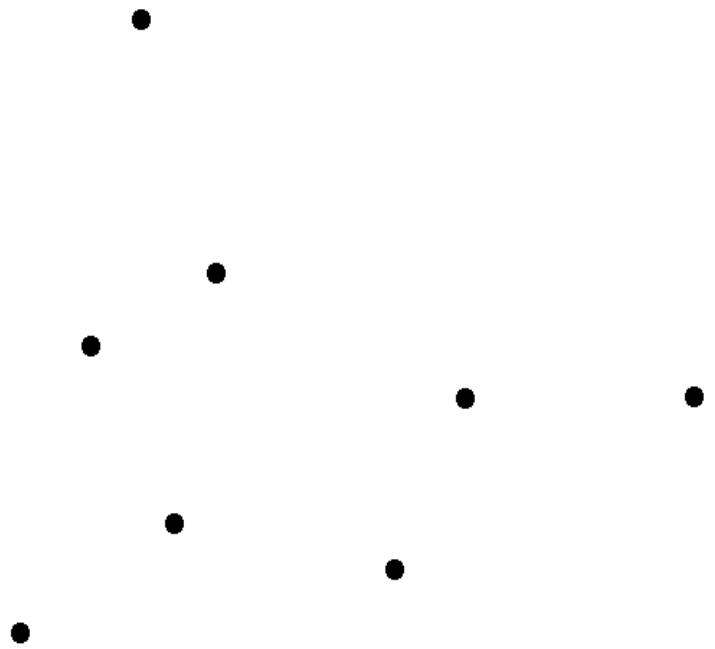



**Figure 33.6** A set of points $Q = \{p_0, p_1, \ldots, p_{12}\}$ with its convex hull $\mathrm{CH}(Q)$ in gray.

# Graham's Scan

- Start at point guaranteed to be on the hull. (the point with the minimum y value)
- Sort remaining points by polar angles of vertices relative to the first point.
- Go through sorted points, keeping vertices of points that have left turns and dropping points that have right turns.

# Graham’s Scan



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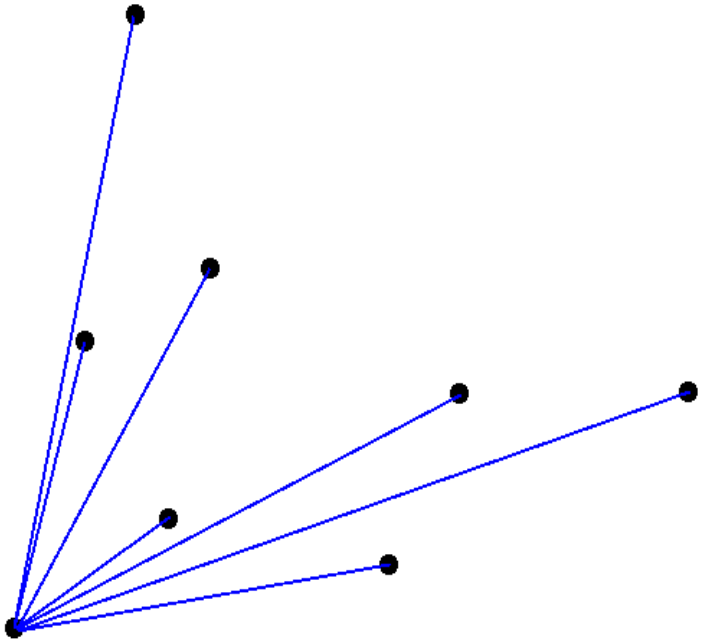

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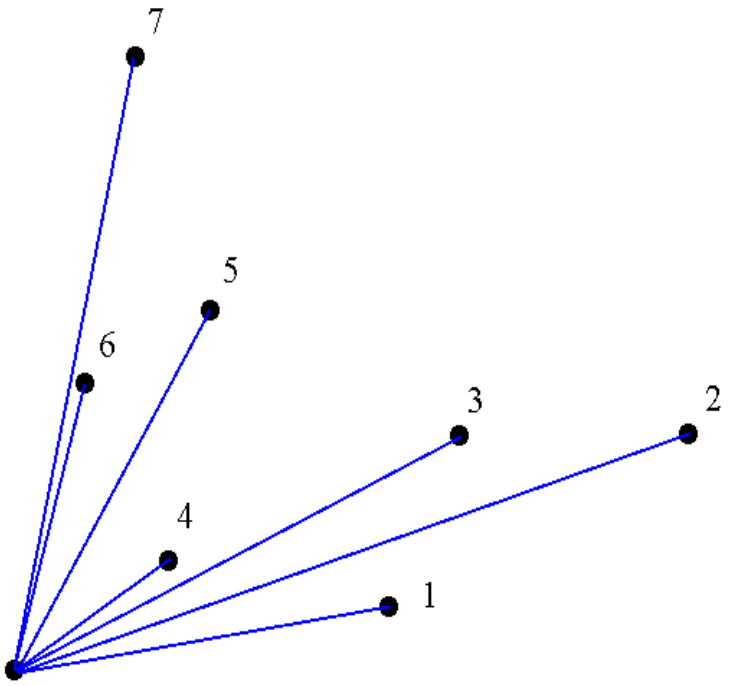

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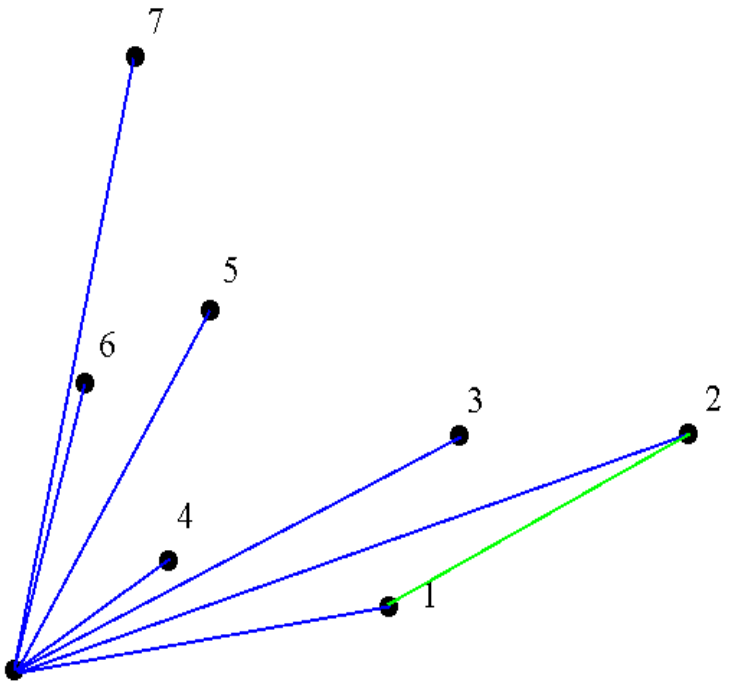

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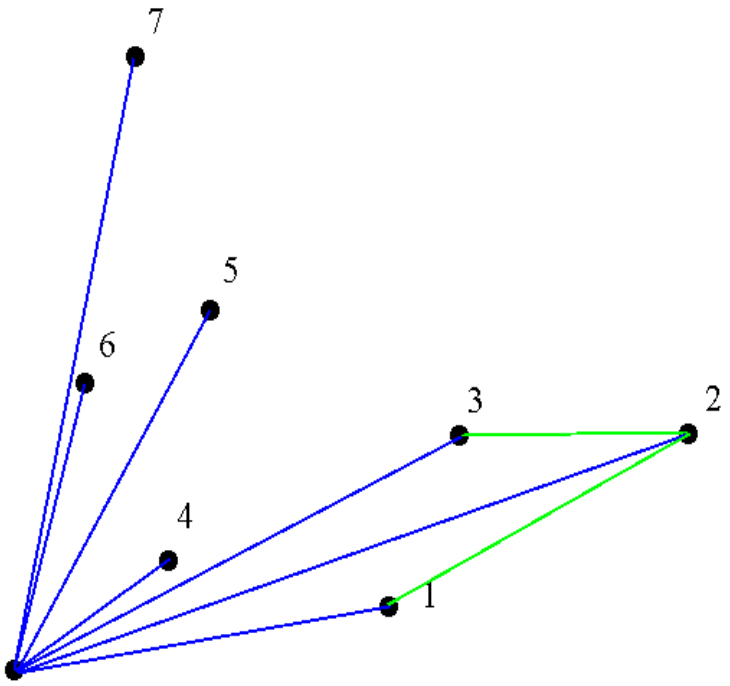

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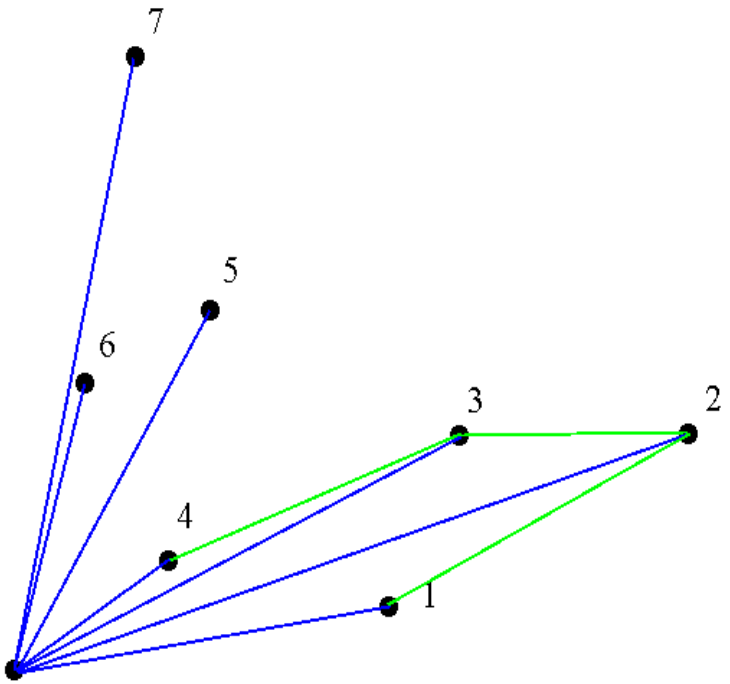

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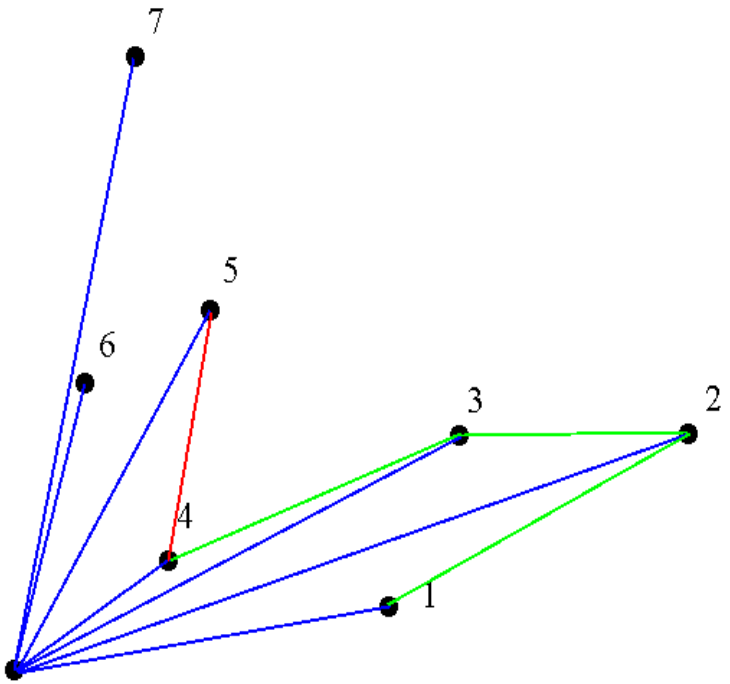

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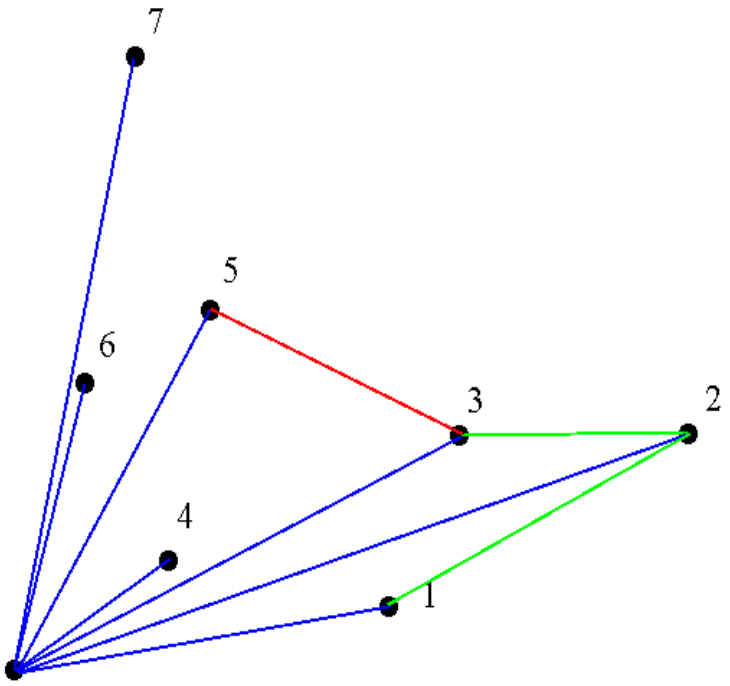

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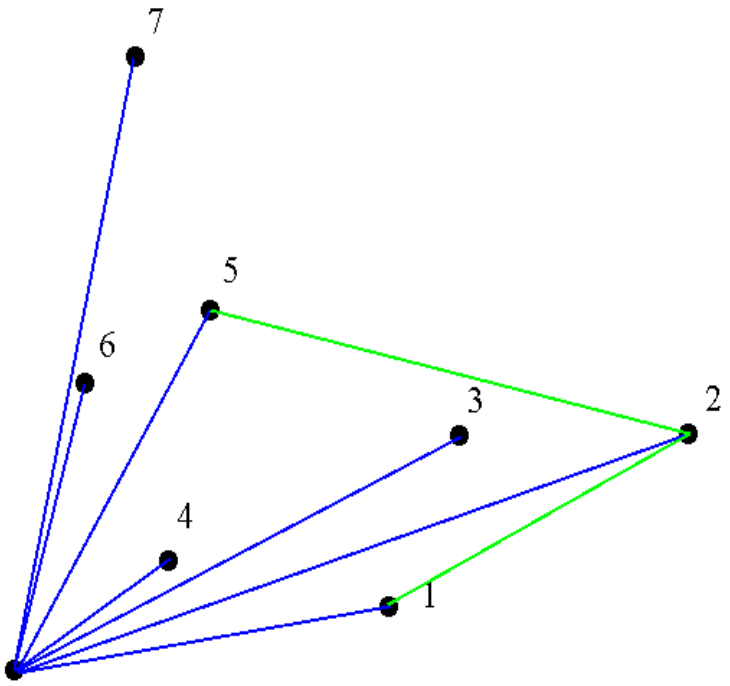

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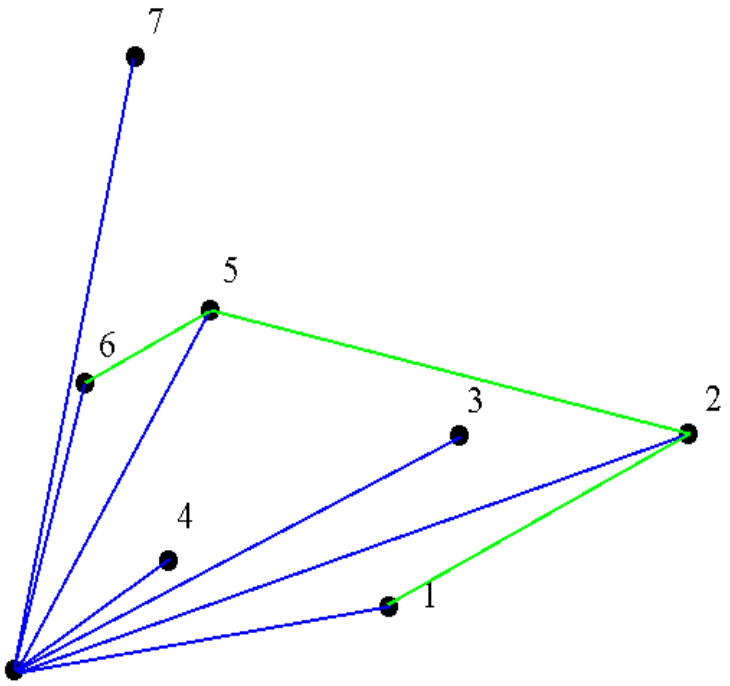

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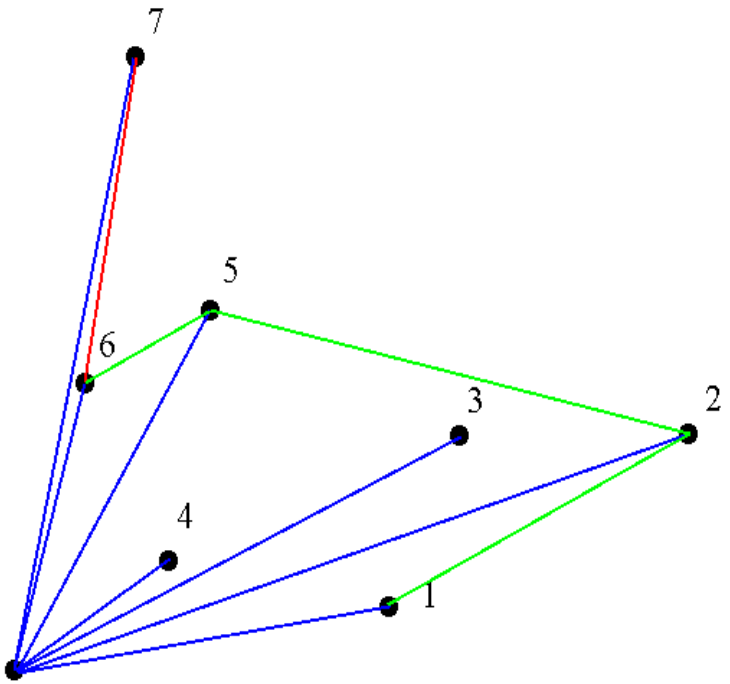

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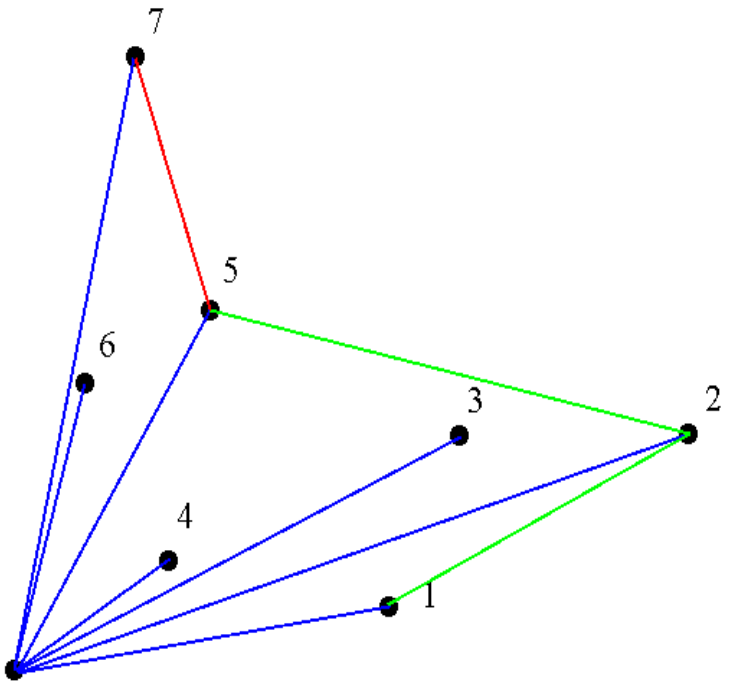

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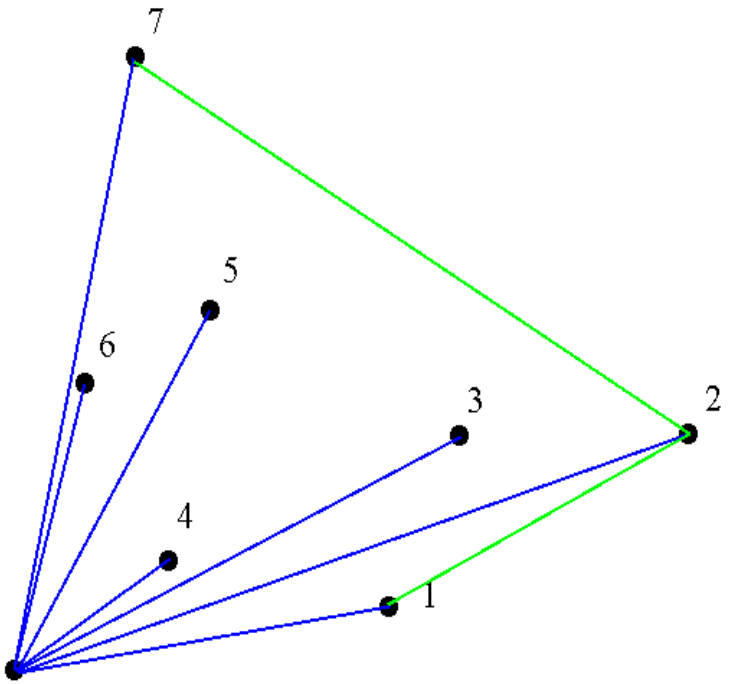

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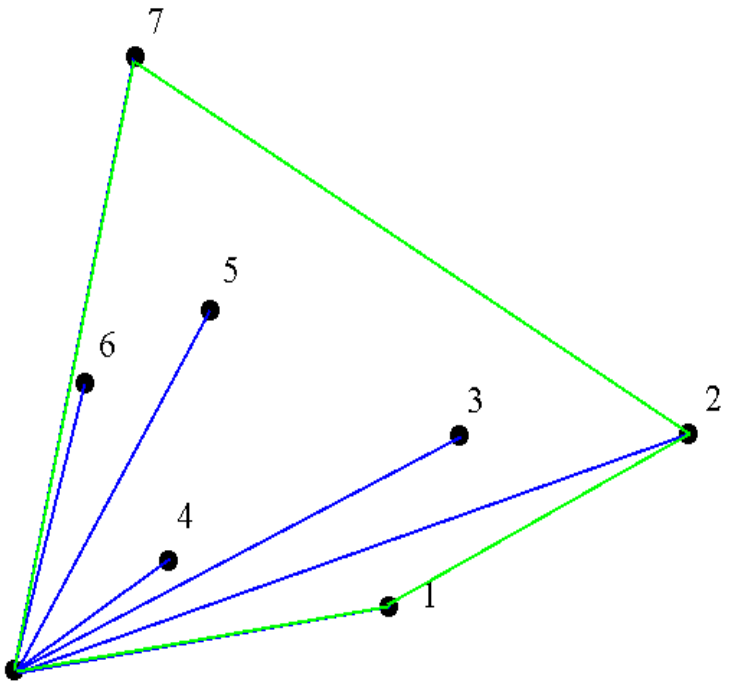


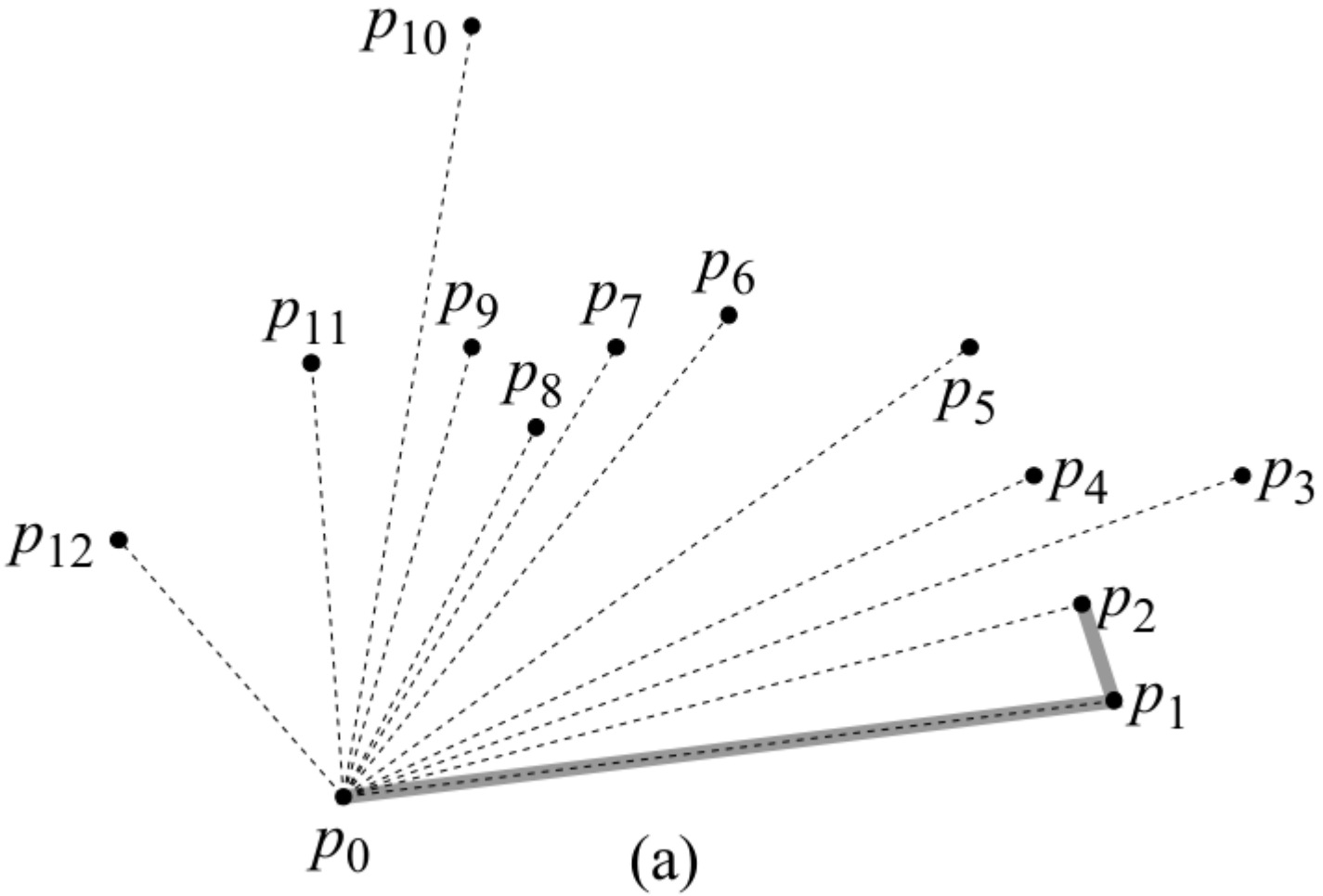
# Graham's Runtime

- Graham's scan is O(n log n) due to initial sort of angles.

## A more detailed algorithm

GRAHAM-SCAN($Q$)

```
1   let p_0 be the point in Q with the minimum y-coordinate,
          or the leftmost such point in case of a tie
2   let ⟨p_1, p_2, ..., p_m⟩ be the remaining points in Q,
          sorted by polar angle in counterclockwise order around p_0
          (if more than one point has the same angle, remove all but
          the one that is farthest from p_0)
3   PUSH(p_0, S)
4   PUSH(p_1, S)
5   PUSH(p_2, S)
6   for i ← 3 to m
7       do while the angle formed by points NEXT-TO-TOP(S), TOP(S),
                    and p_i makes a nonleft turn
8           do POP(S)
9           PUSH(p_i, S)
10  return S
```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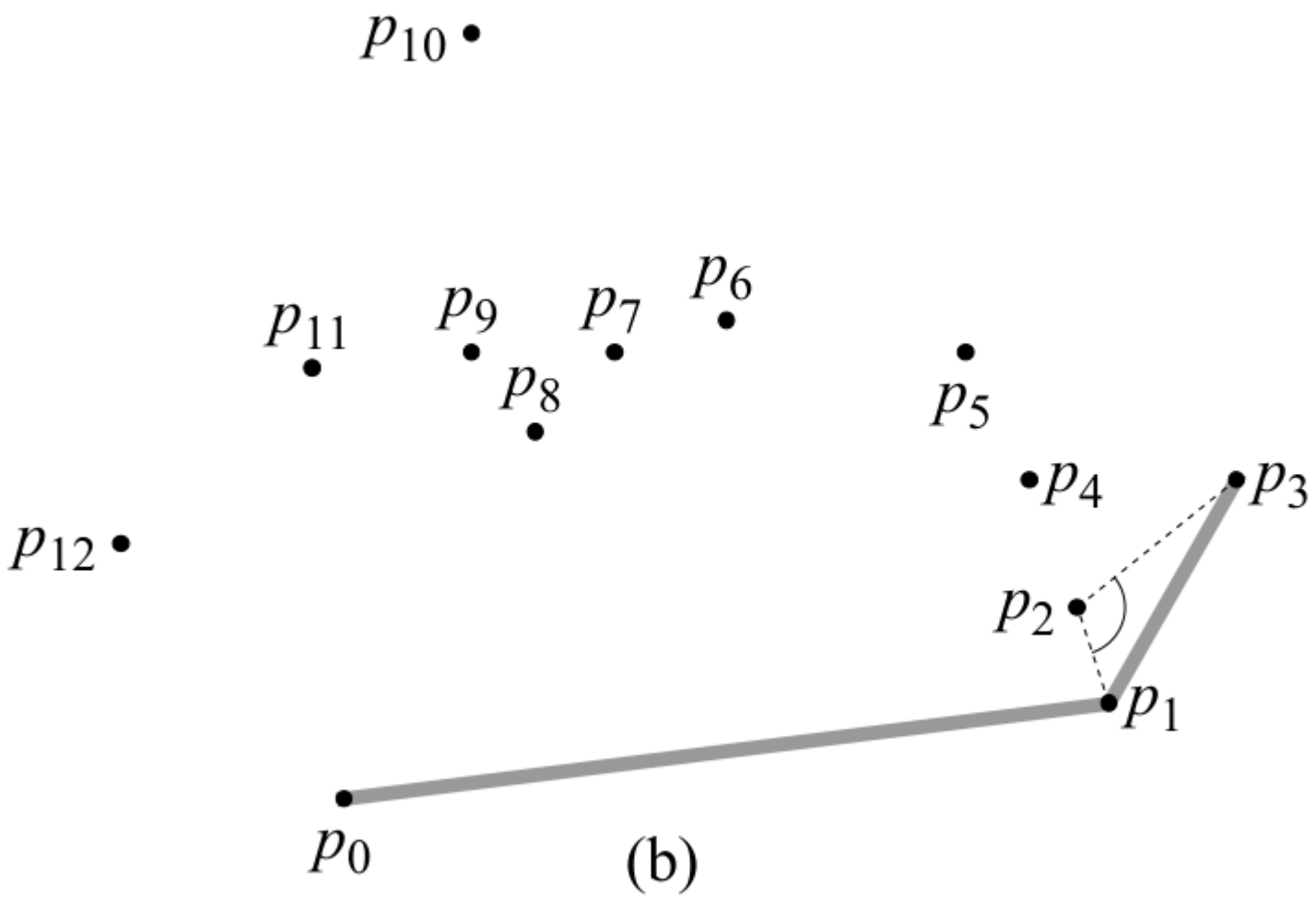
(a)


$p_{10}$
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$p_2$
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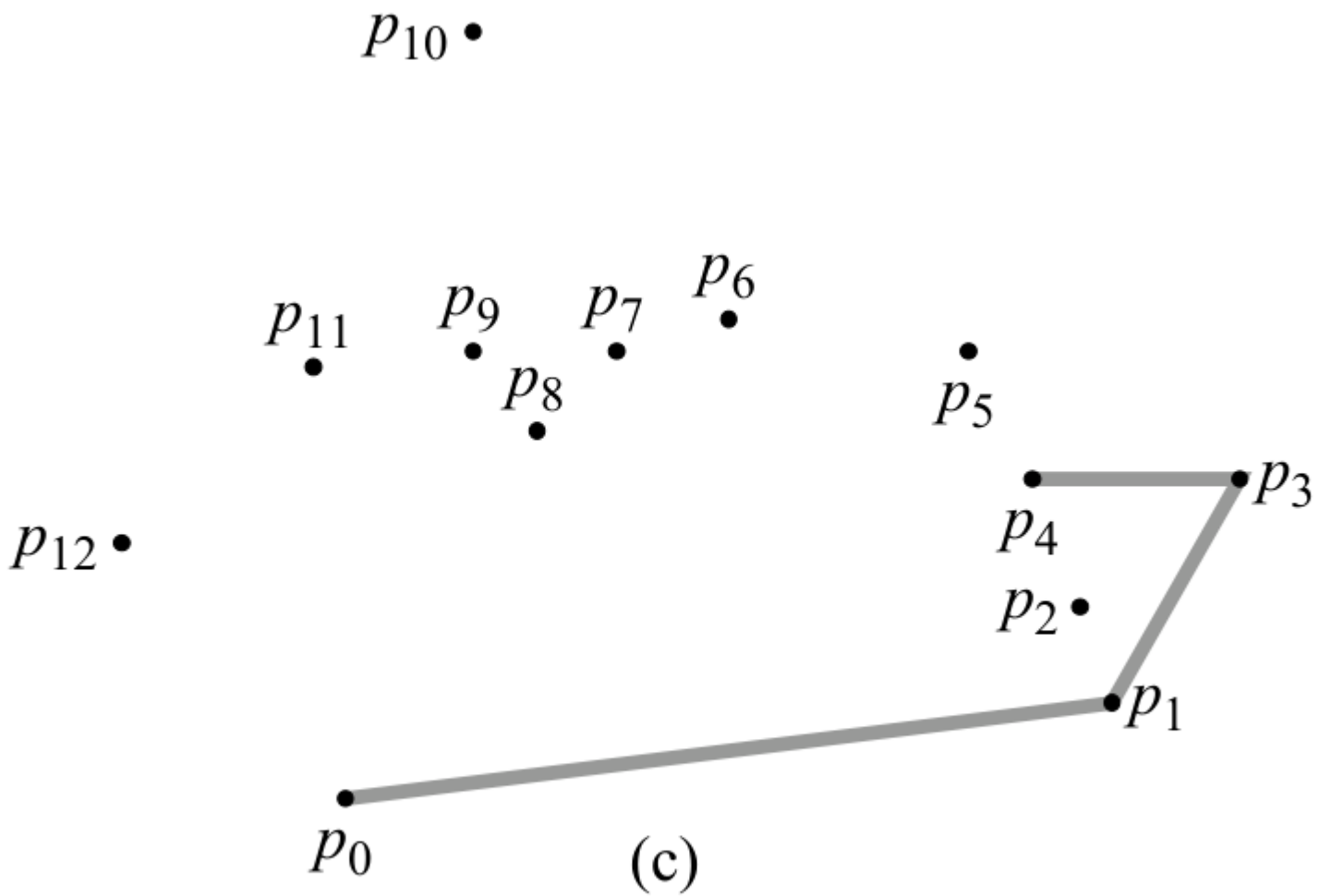
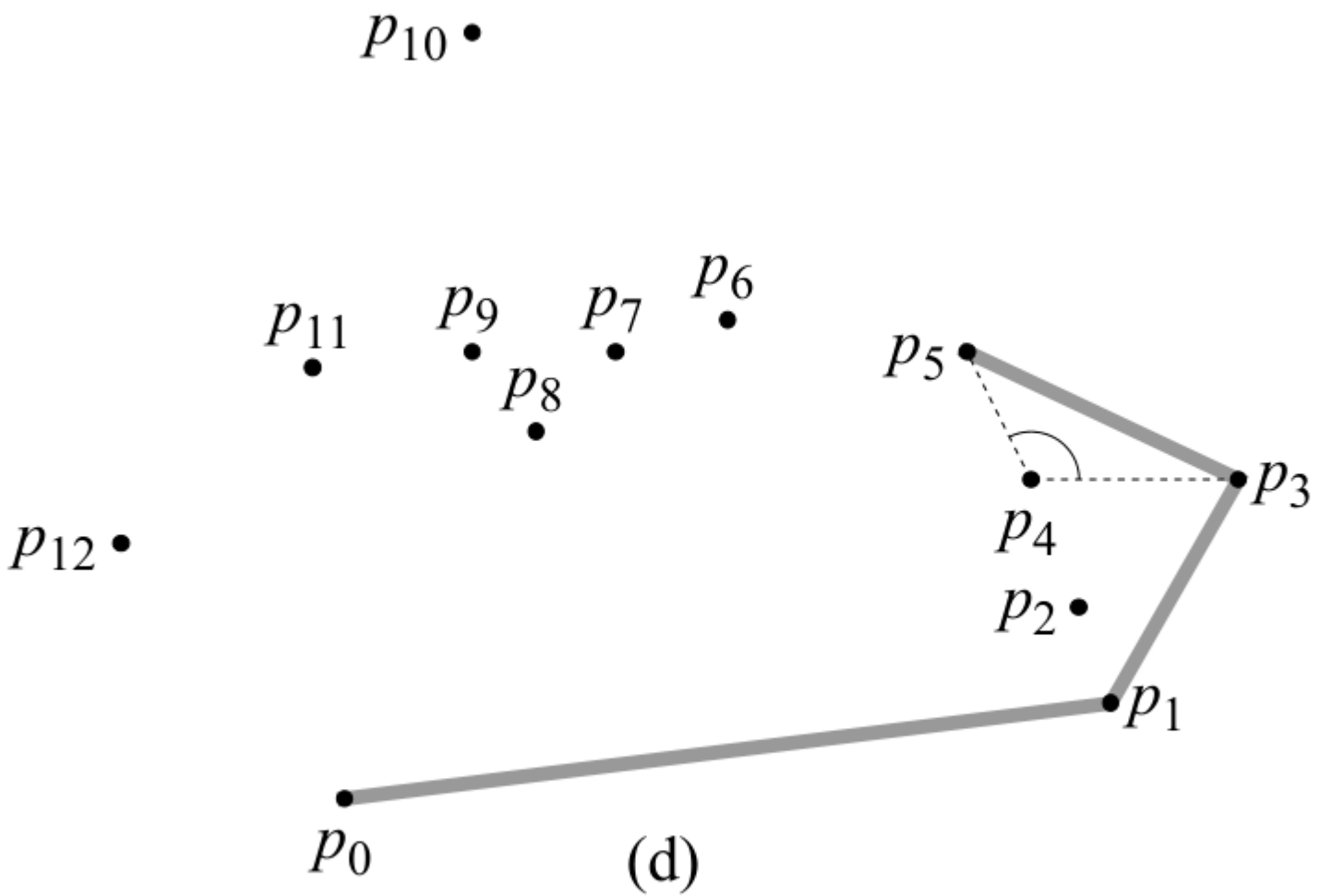
$p_0$
(b)




(c)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$p_4$
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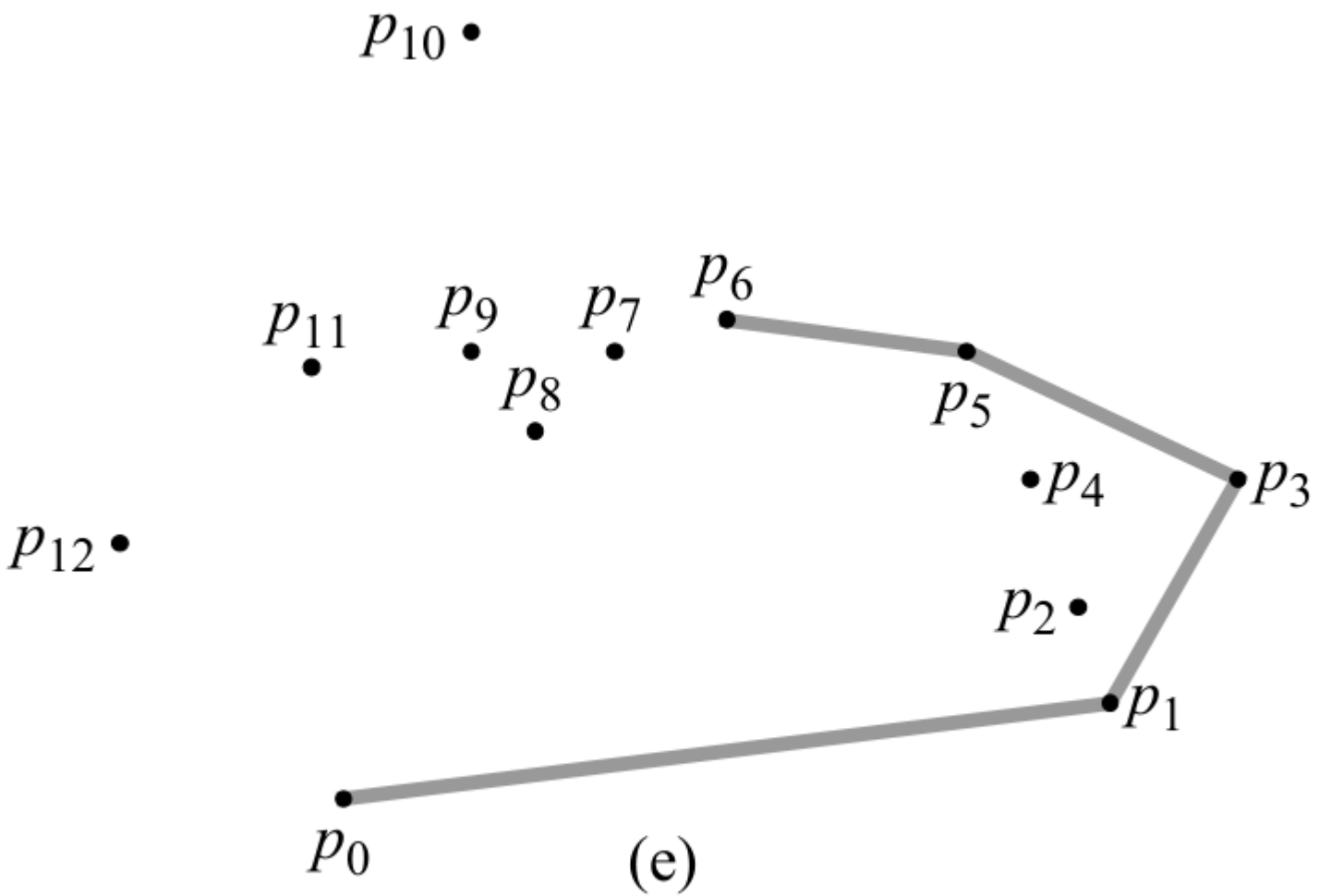
(d)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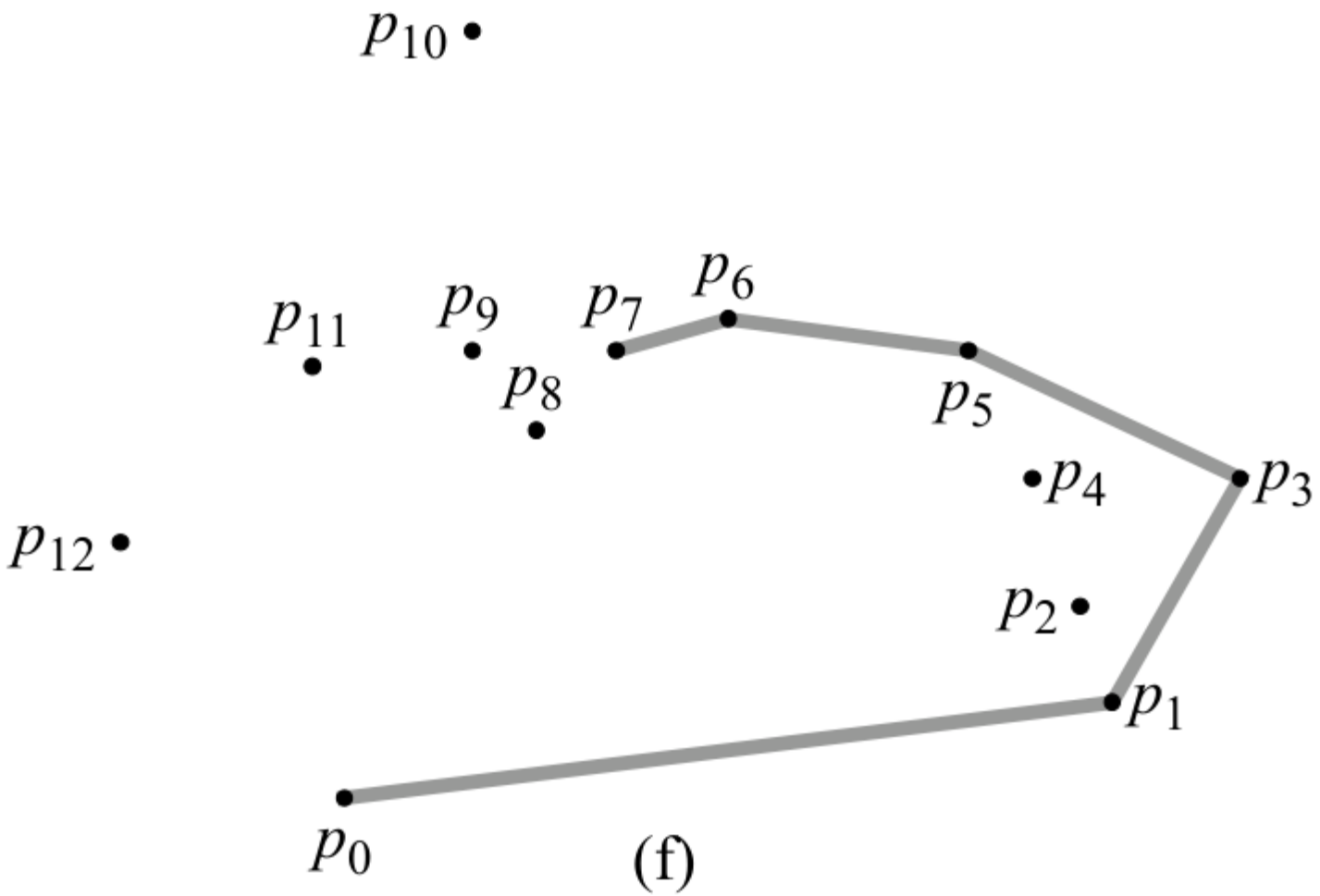
(e)

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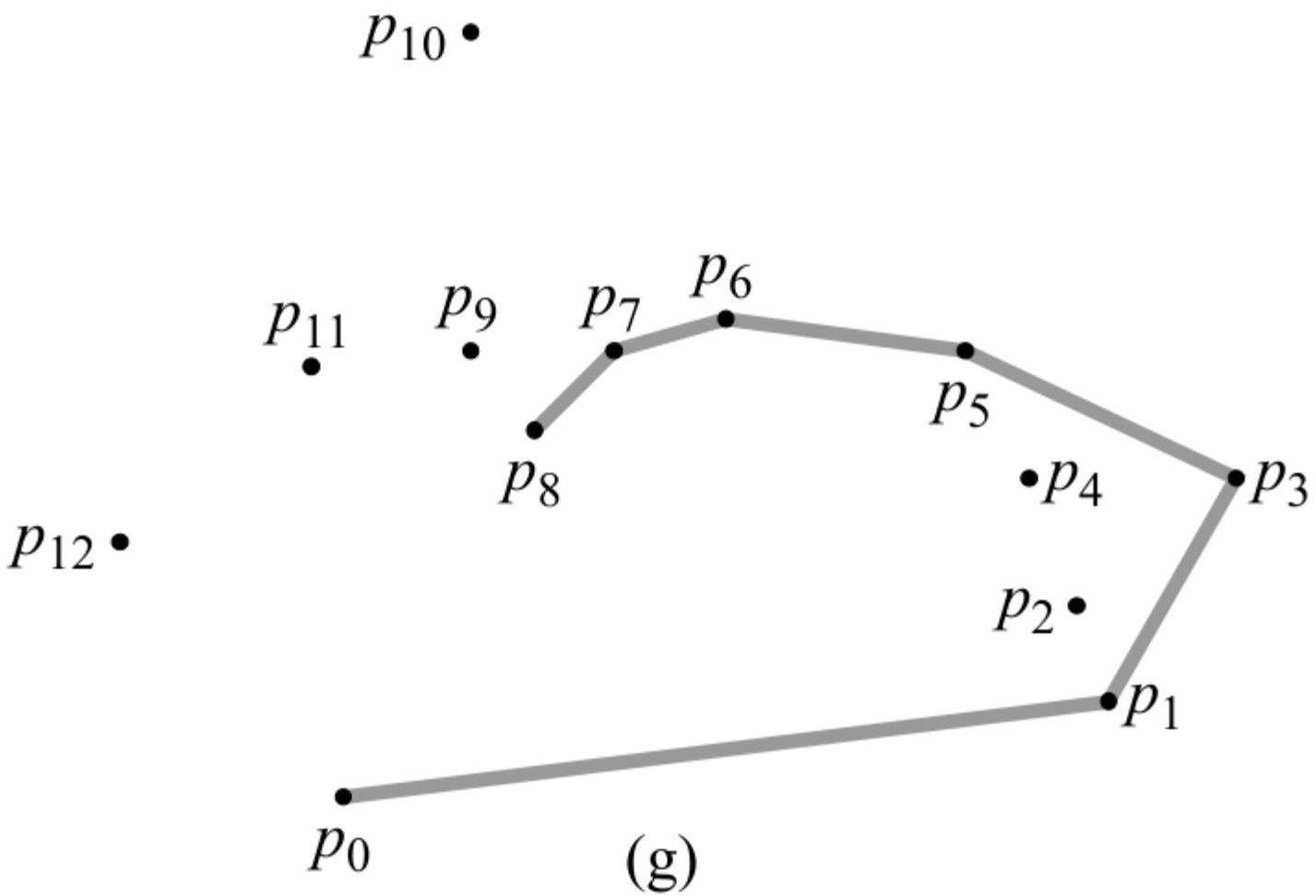


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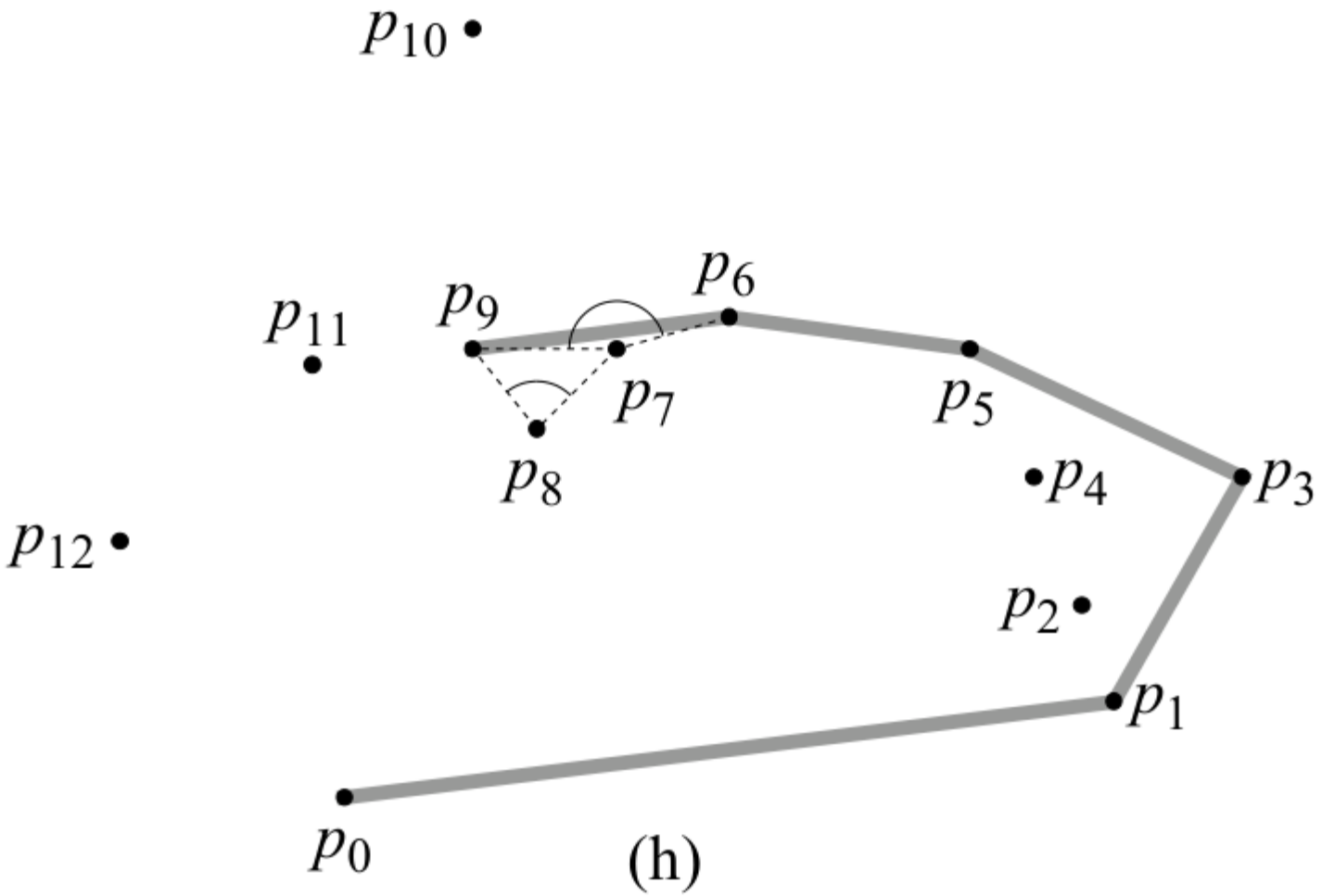


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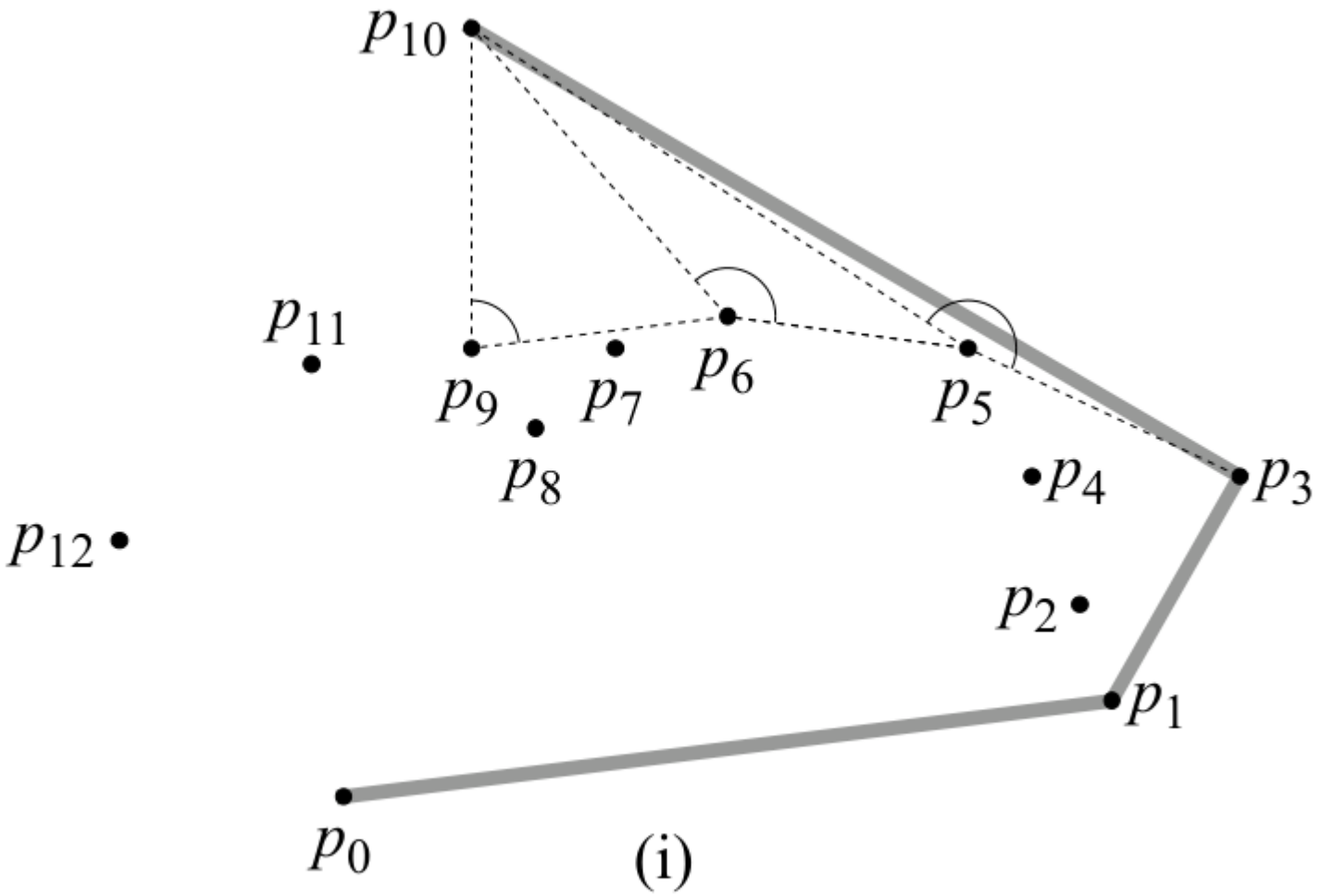
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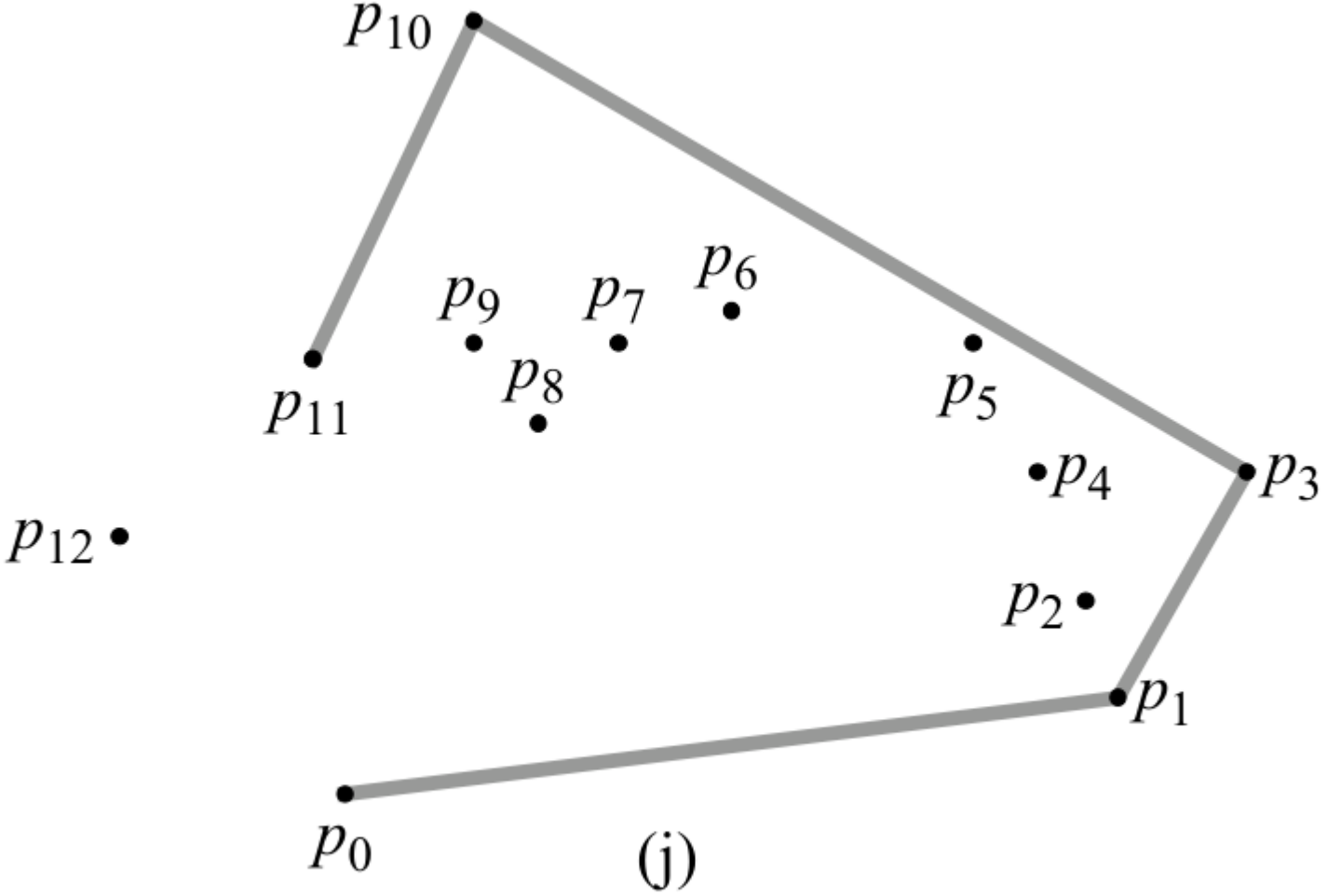
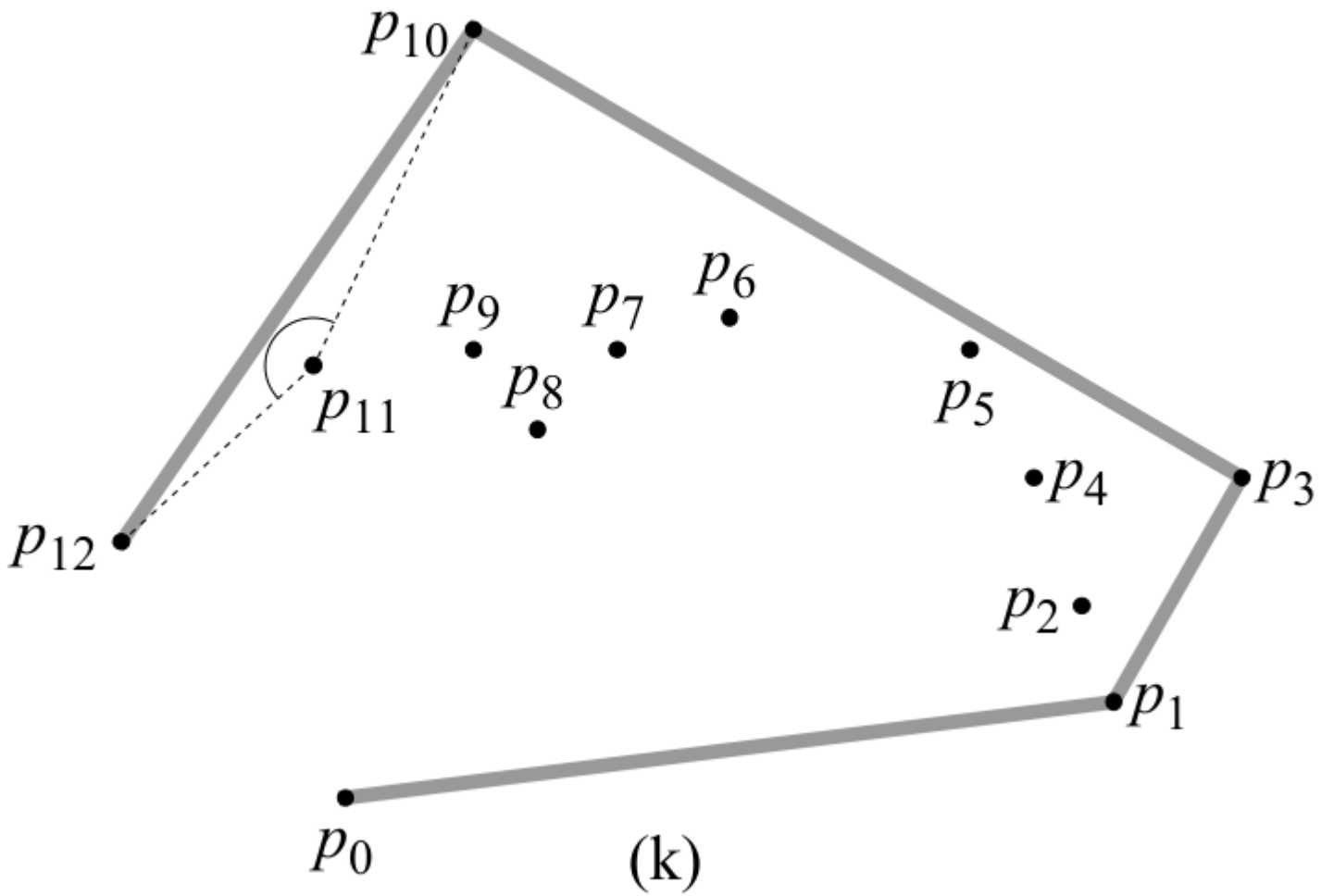
(i)




(j)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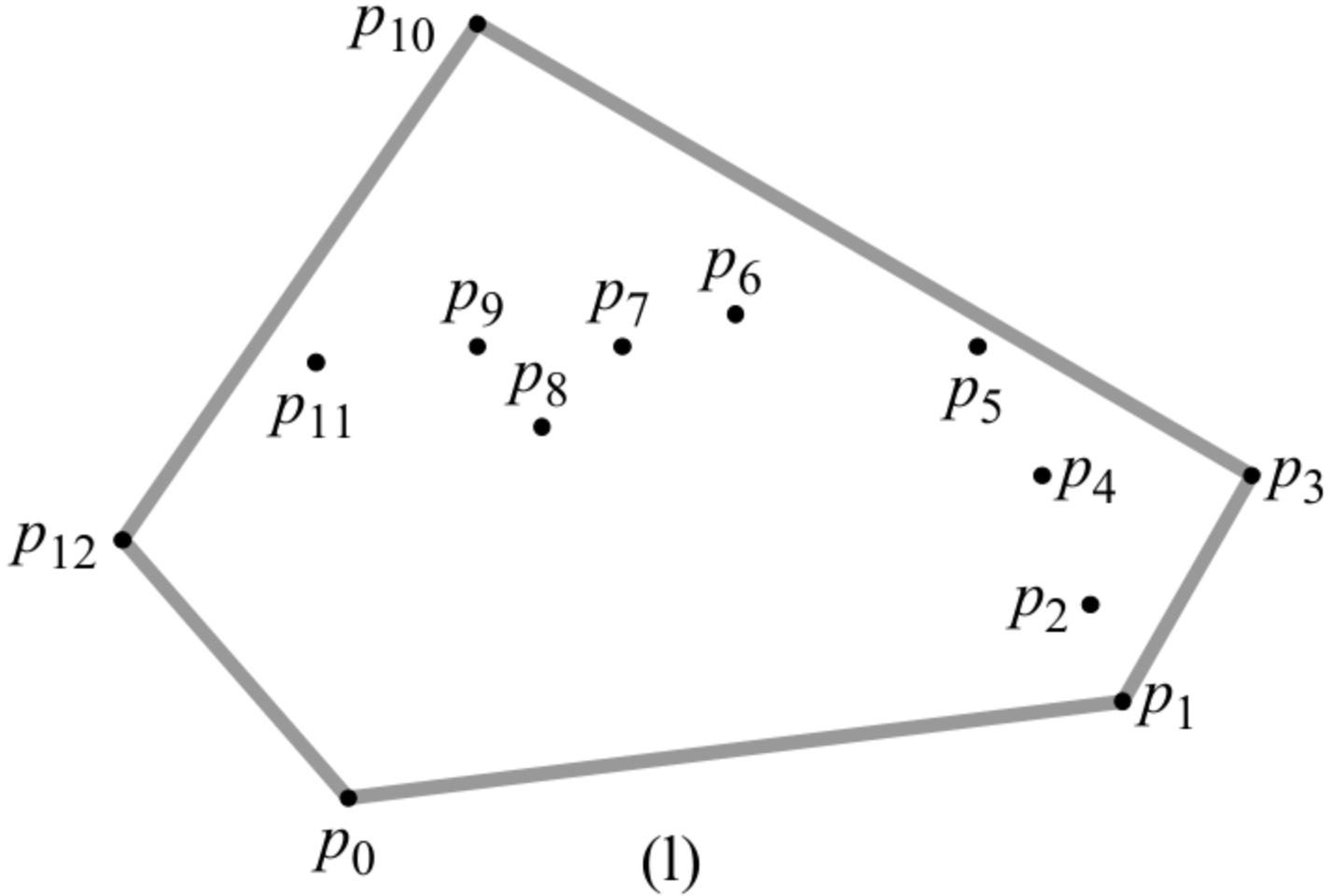
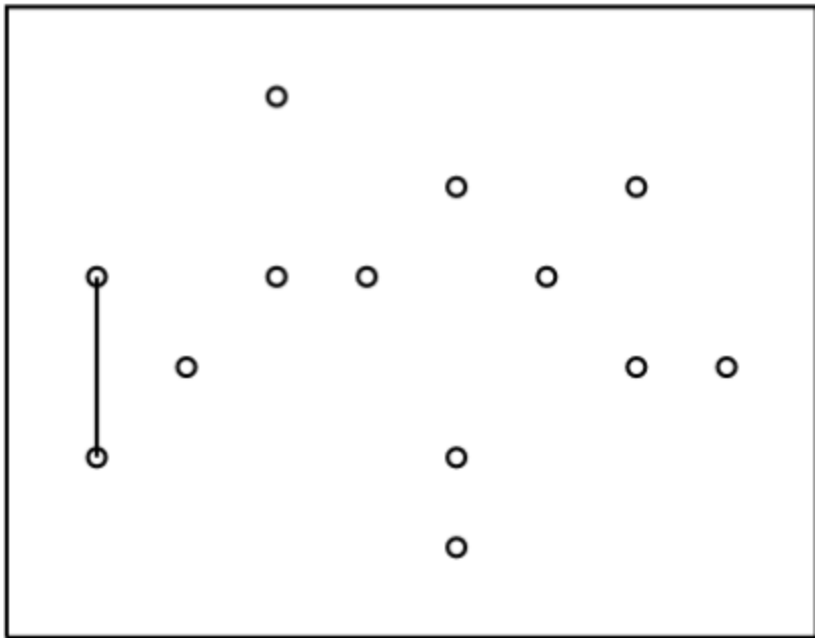
(k)

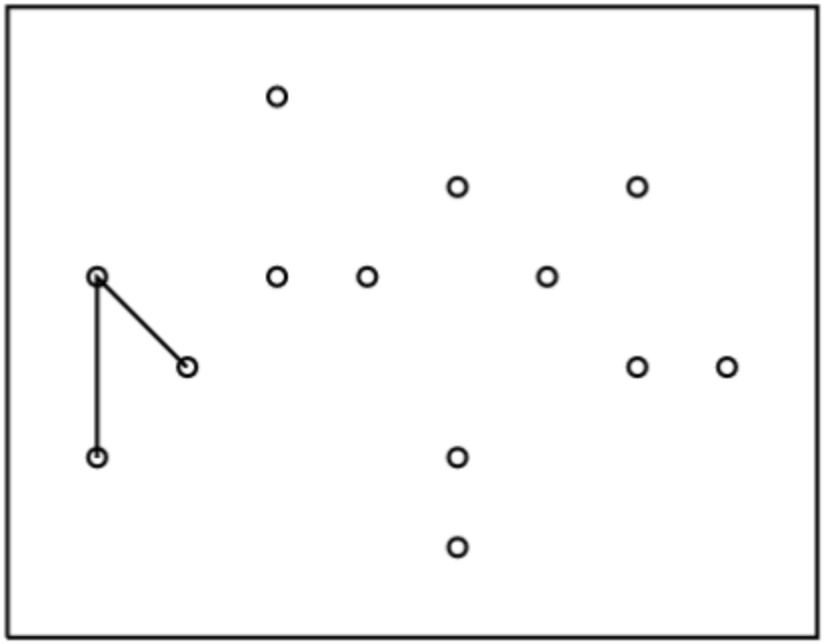


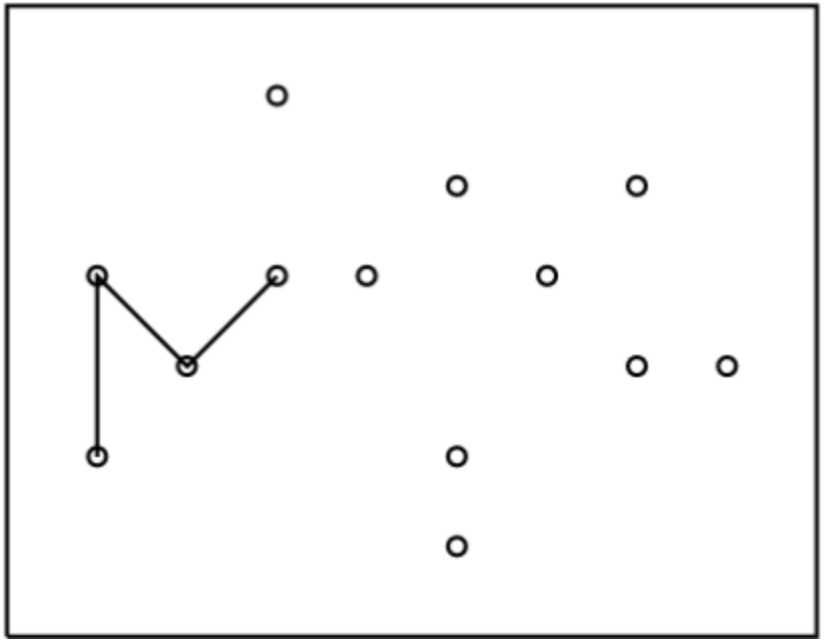

(l)

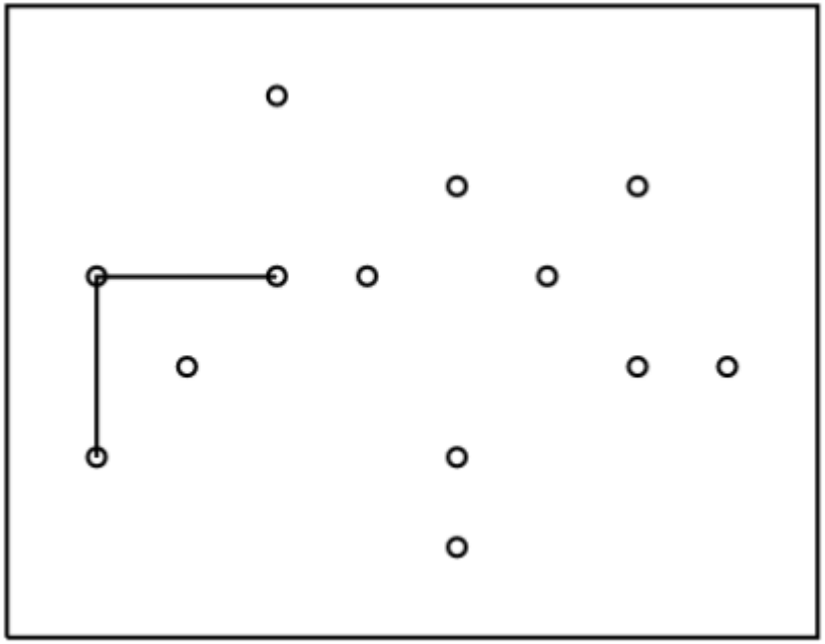
# Andrew's algorithm

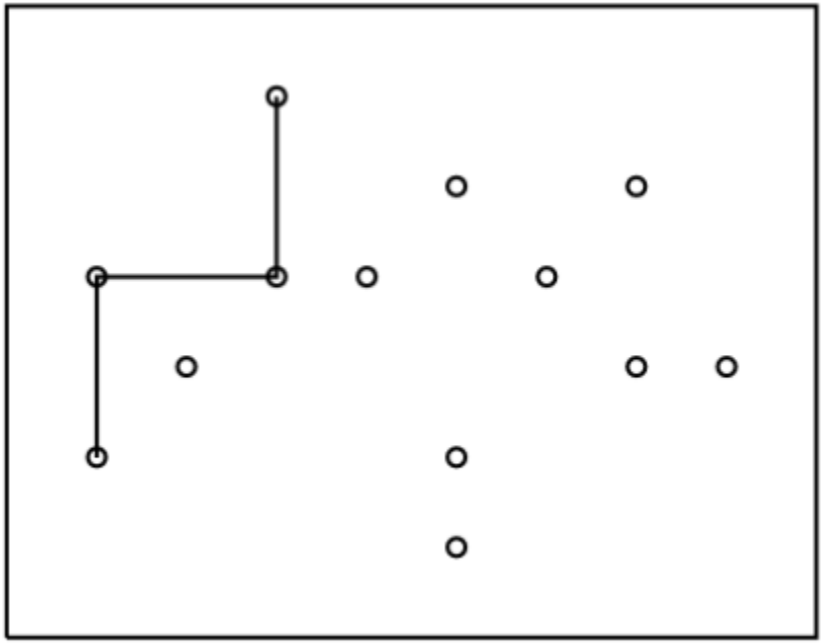
- The algorithm first determines the leftmost and rightmost points in the set, and then constructs the convex hull in two parts: first the upper hull and then the lower hull.
- Both parts are similar, so we can focus on constructing the upper hull.
- First, we sort the points primarily according to x coordinates and secondarily according to y coordinates

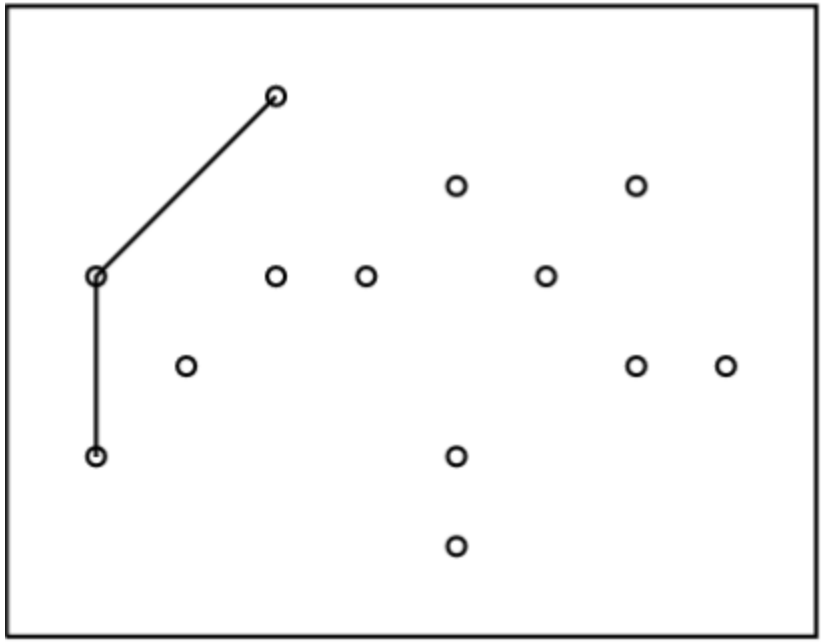


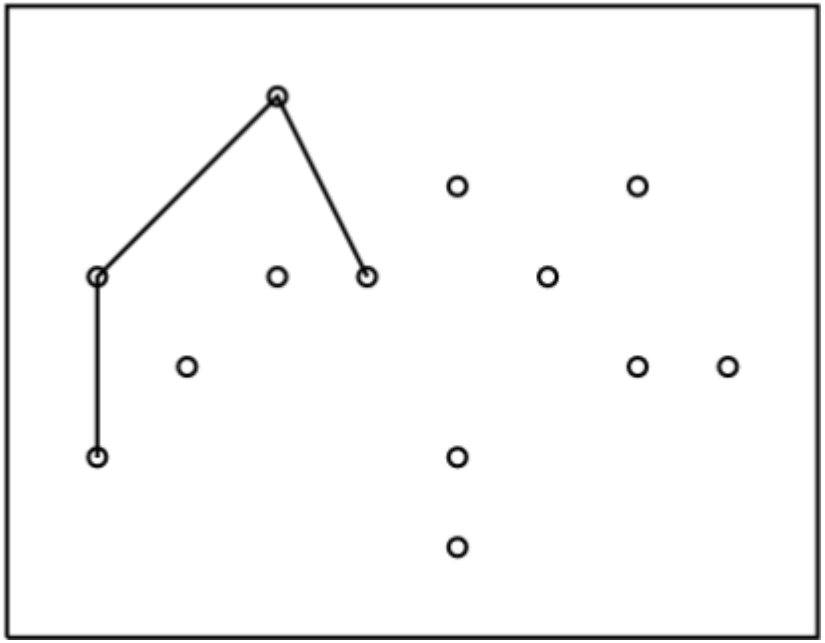


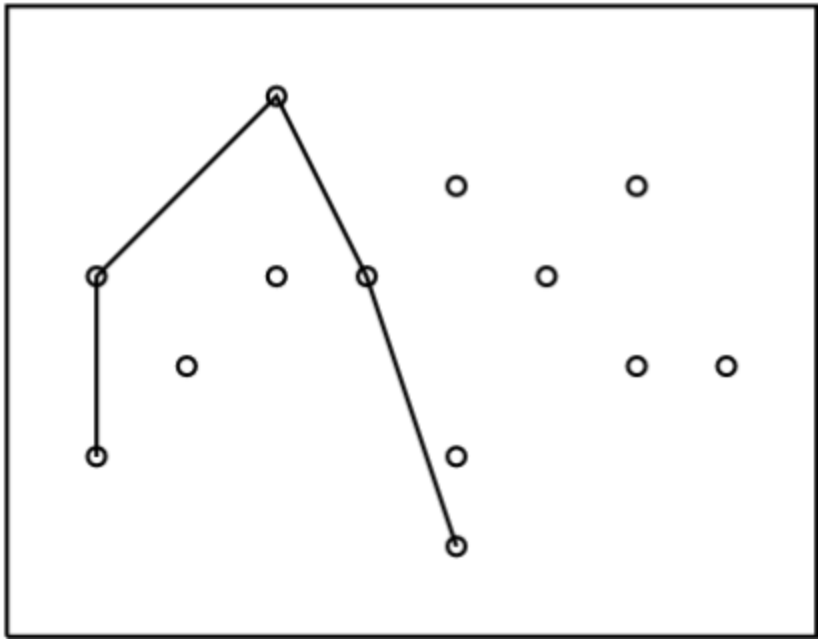


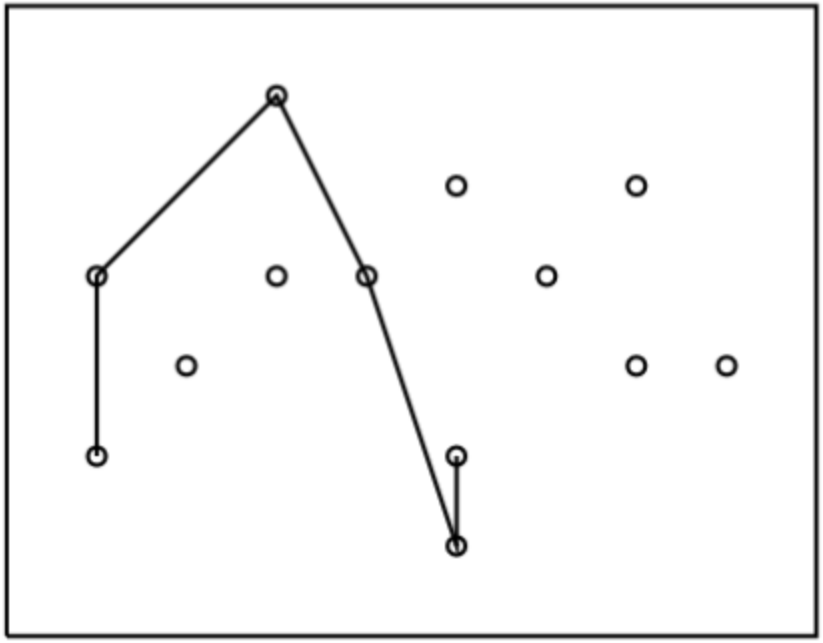


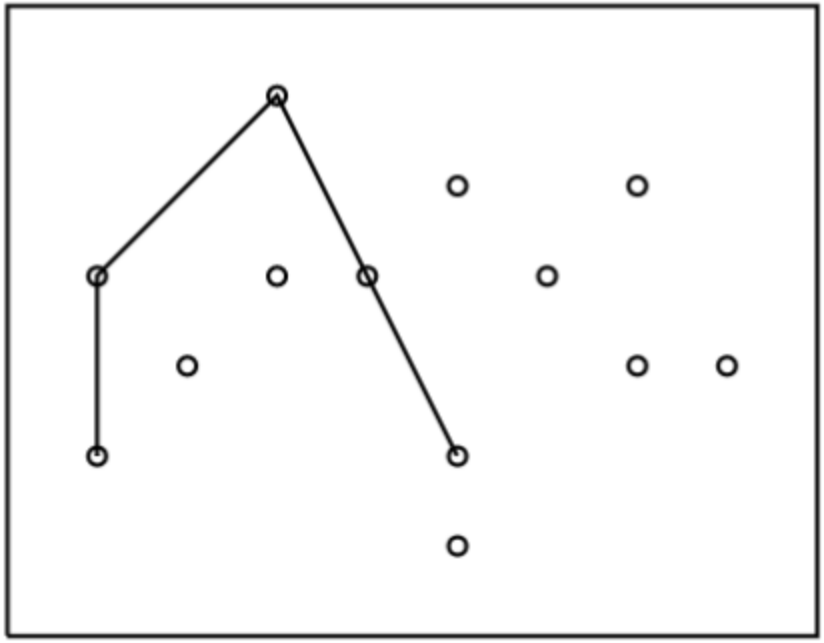


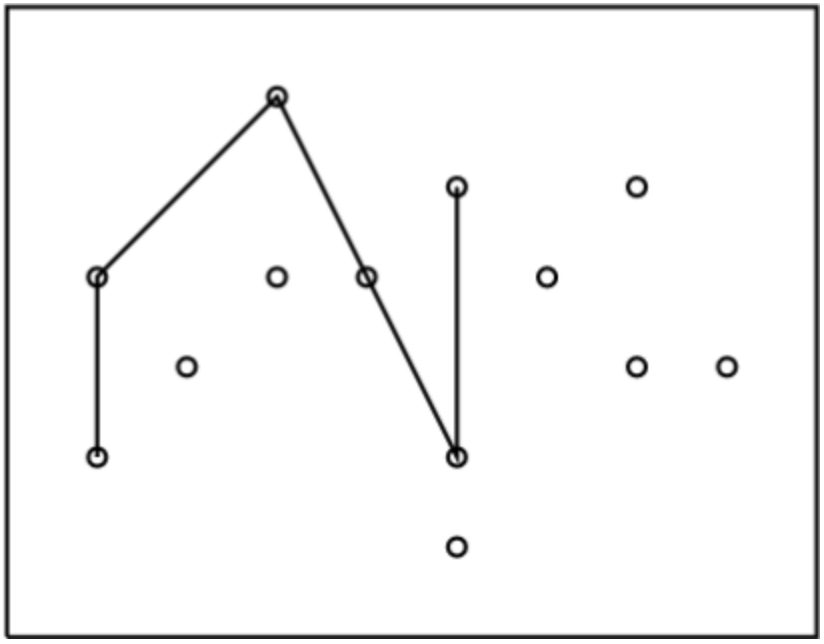


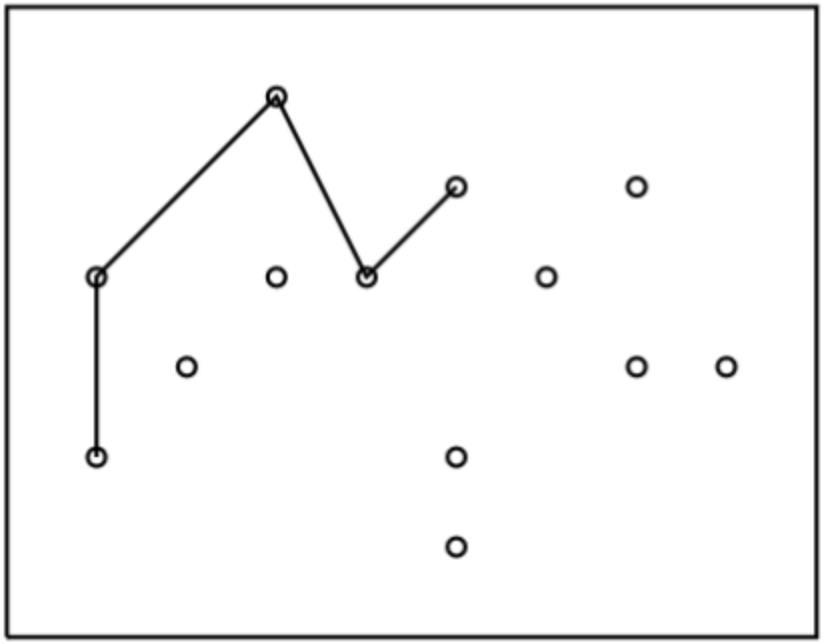


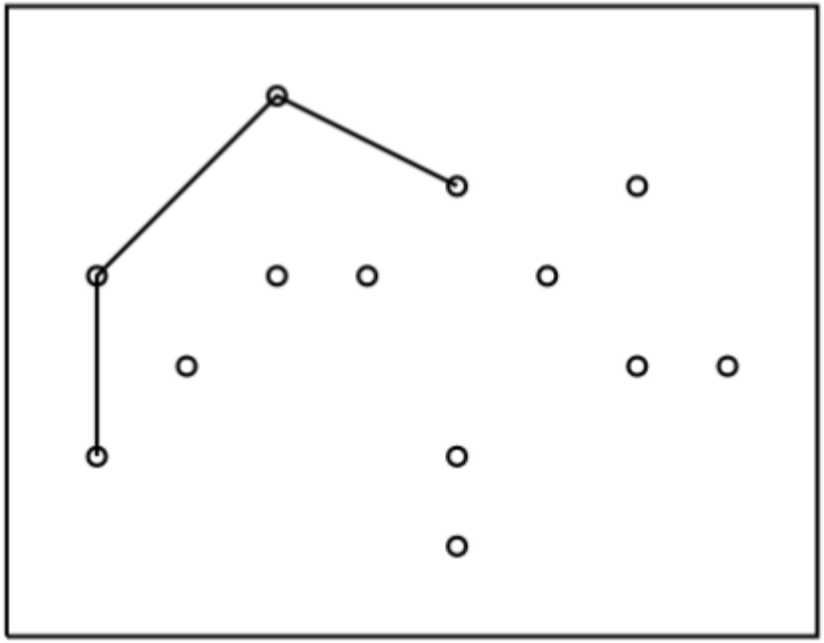


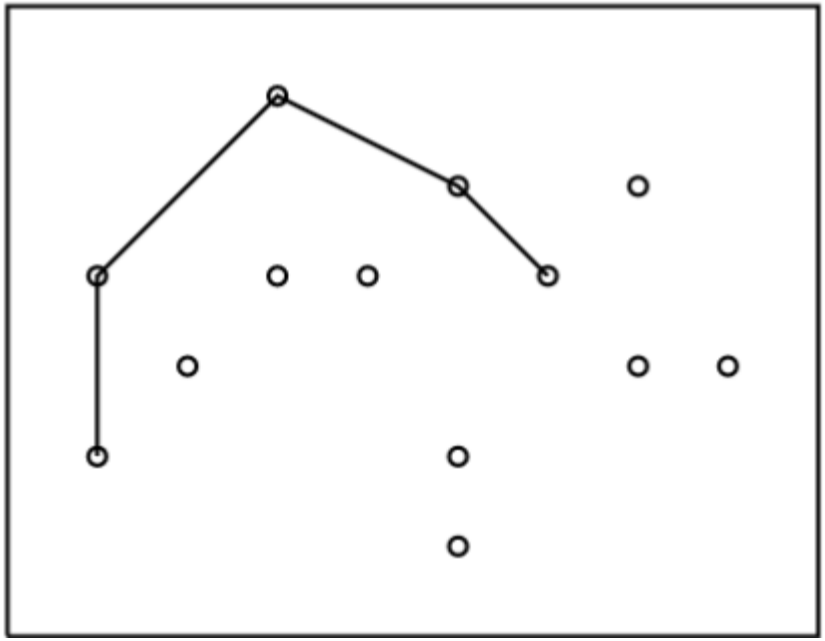


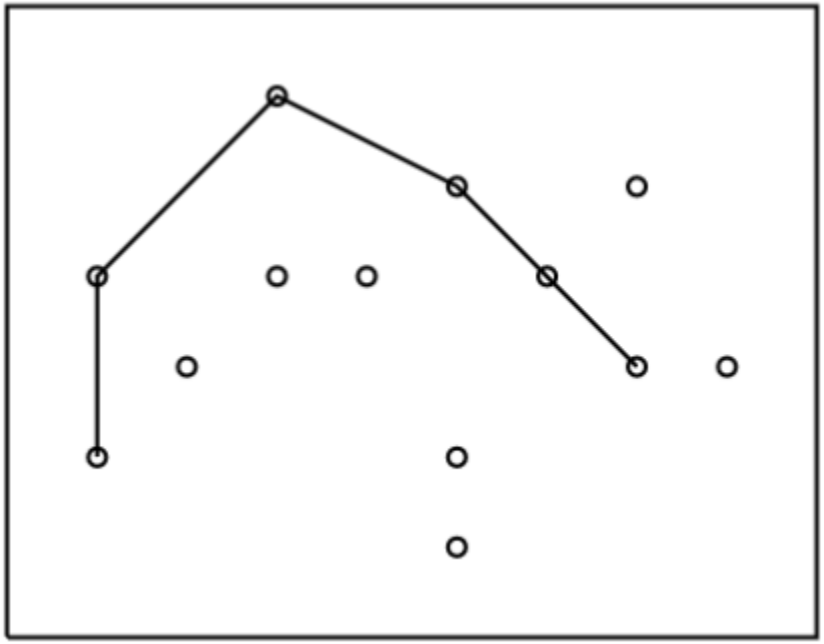


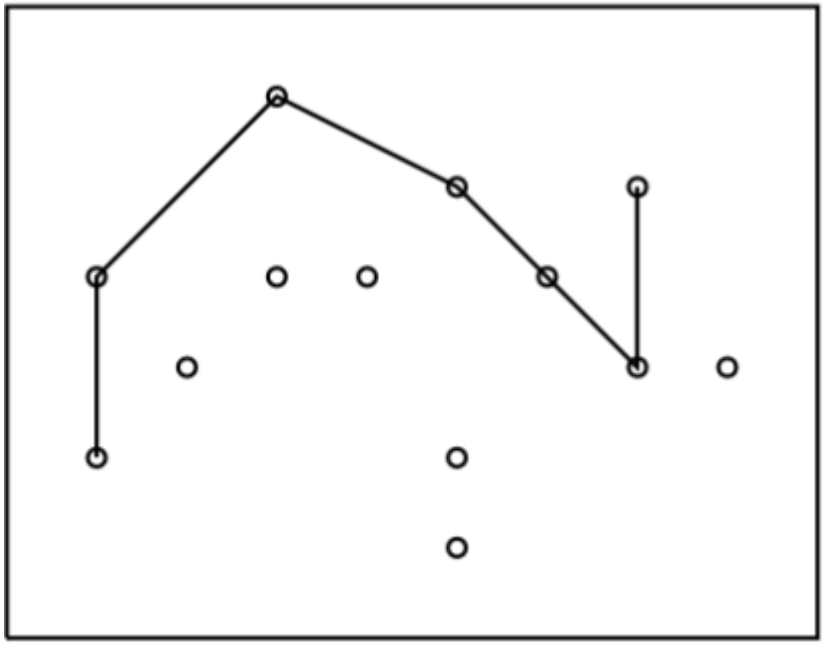


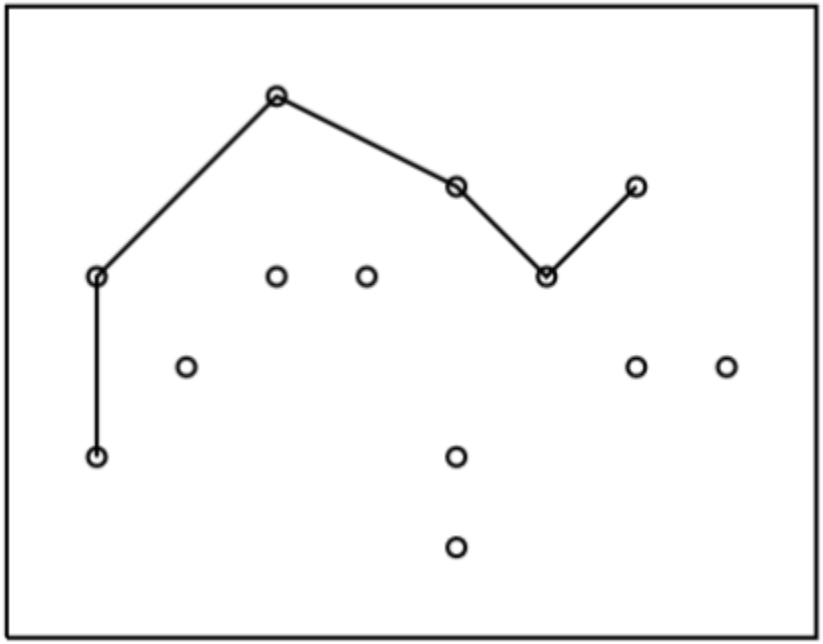


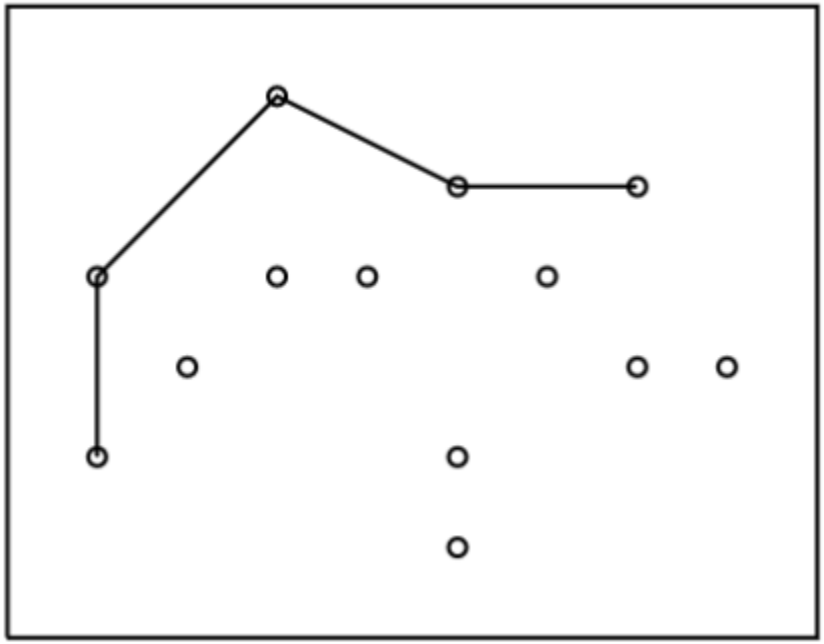


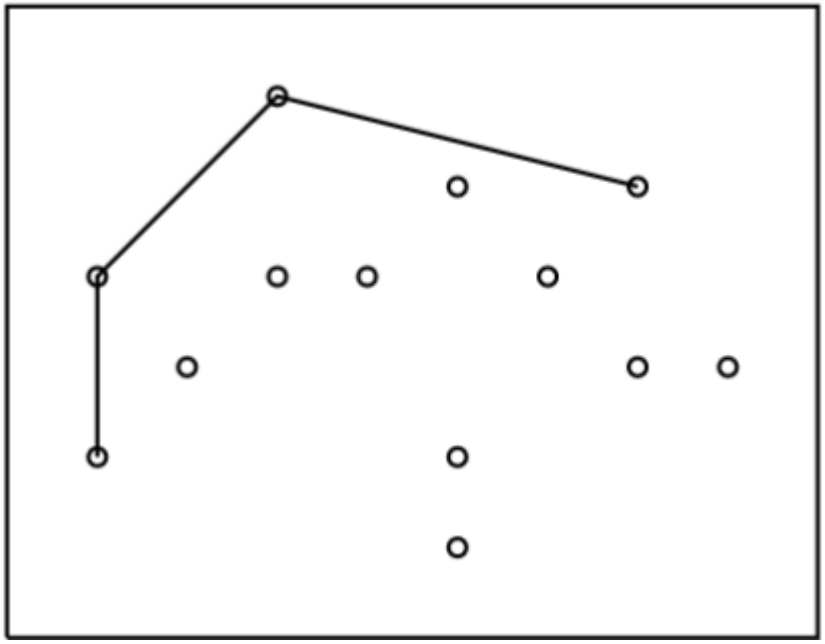


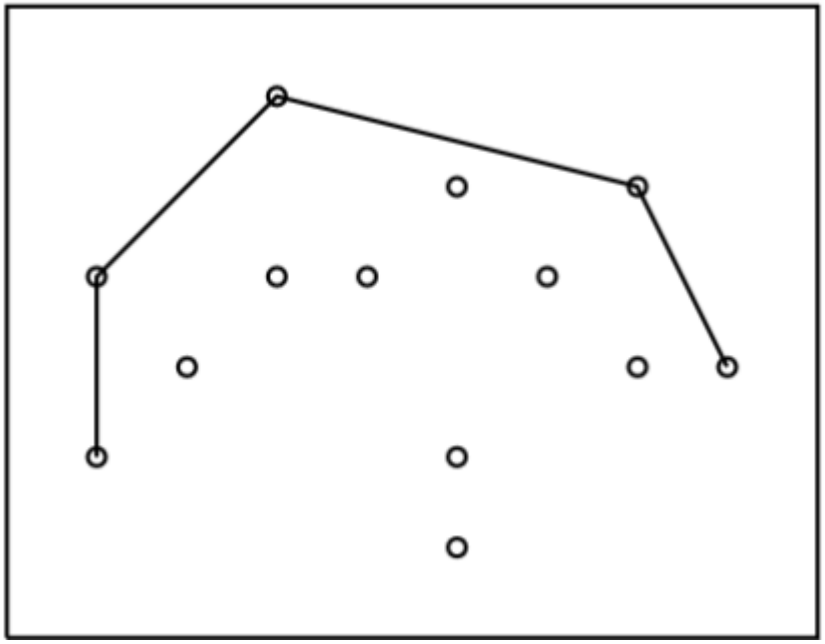


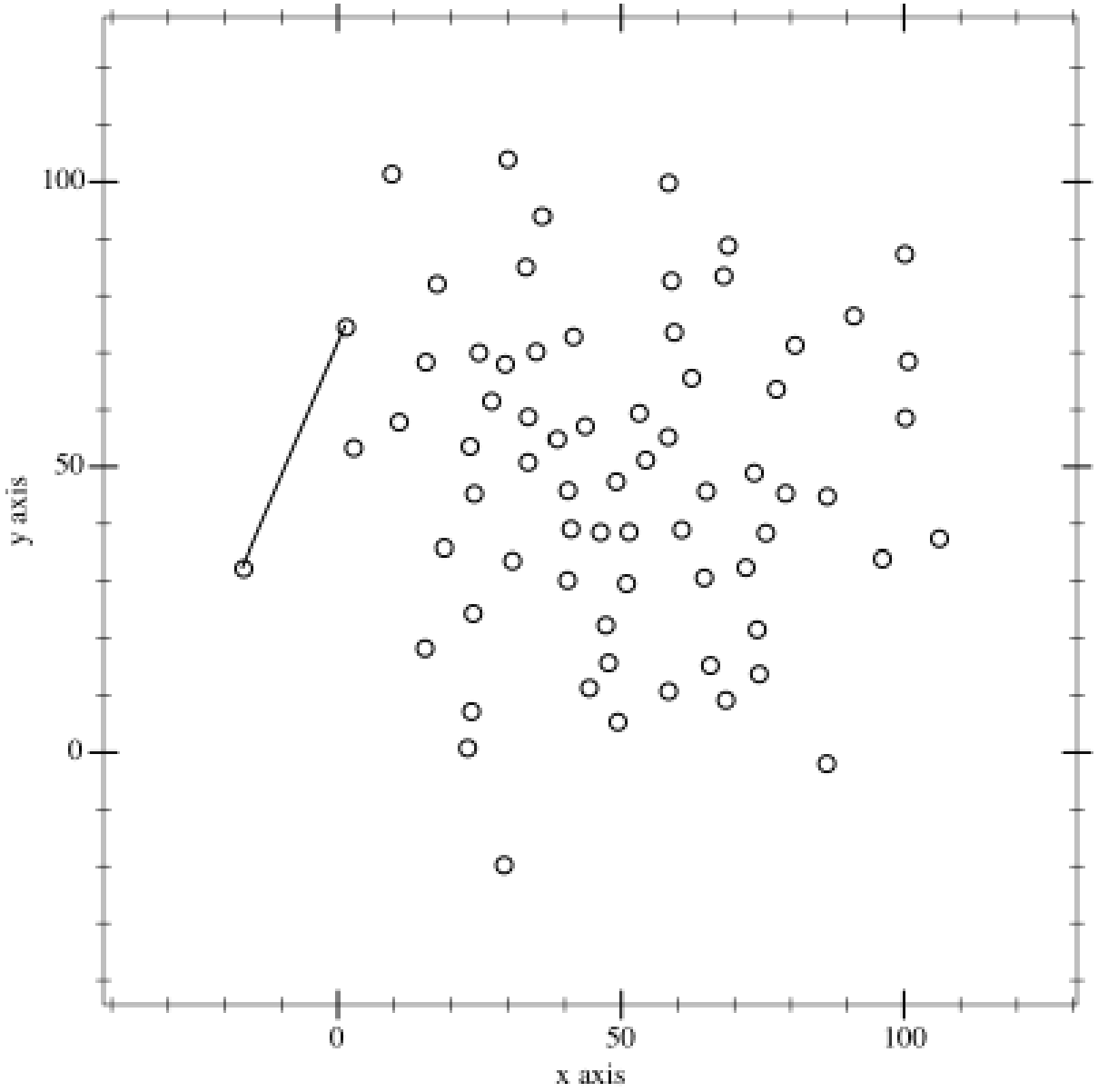








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# Bibliografia


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